

# Multivariate GARCH with Only Univariate Estimation

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## Abstract

This brief note offers an explicit algorithm for a multivariate GARCH model, called PC-GARCH, that requires only univariate GARCH estimation. It is suitable for problems with hundreds or even thousands of variables. PC-GARCH is compared to two other techniques of getting multivariate GARCH using univariate estimates.

## 1 Introduction

Unfortunately the availability of multivariate GARCH software is limited even though it could be extremely useful for risk management and other applications. The leading multivariate models are BEKK [Engle and Kroner, 1995] ([Engle and Mezrich, 1996] is a more applied look at the model), and Dynamic Conditional Correlations [Engle, 2002], [Engle and Sheppard, 2001].

However, it is possible to get multivariate estimates by only using univariate GARCH estimates. [Alexander, 2000] describes one such scheme called Orthogonal GARCH. [Harris et al., 2004] presents a different method. Another, referred to as PC-GARCH, is outlined here. These three approaches are compared in Section 4.

## 2 The PC-GARCH Algorithm

The algorithm is broken into several steps. The input is a matrix of (log) returns with rows representing  $T$  points in time and columns representing  $N$  variables (often assets). The return of variable  $i$  at time  $t$  is denoted  $x_{ti}$ .

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\*This report can be found in the working papers section of the Burns Statistics website <http://www.burns-stat.com/>.

### Step 1

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Estimate a univariate GARCH model on each variable.

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Although the model for each variable could be different, typically the same model will be used for all of the variables. The parameters are optimized independently for each variable. Note that not all GARCH models are equally good—see for example [Burns, 2002].

### Step 2

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Standardize the residuals with the estimated variance and mean processes for each variable.

The standardized residuals are created by

$$r_{ti} = (x_{ti} - m_i) / s_{ti} \quad (1)$$

where  $m_i$  is the estimated mean for variable  $i$ , and  $s_{ti}$  is the square root of the variance estimated by the GARCH model at time  $t$  for variable  $i$ .

Denote the matrix containing the  $r_{ti}$  as  $R$ .

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While we have assumed in (1) that the estimated mean is constant for the variables, this need not be the case. For example, an autoregressive term for the mean could be used.

If the univariate models were true, then each element of  $R$  would have mean 0 and variance 1. In reality this will be close to the case.

### Step 3

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Perform principal components (an eigen rotation) on the matrix of standardized residuals. We get the principal component matrix  $P$  as

$$P = RL \quad (2)$$

where  $L$  is the matrix of loadings.

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### Step 4

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Estimate a univariate GARCH model for each principal component (that is, for each column of  $P$ ).

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The models for the principal components can most likely be quite simple. Specifically, a GARCH(1,1) is generally recommended.

### Step 5

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Use the loading matrix  $L$  to rotate the principal component variances back to variable space. At each point in time compute:

$$C_t = LV_tL^T \tag{3}$$

where  $V_t$  is the diagonal matrix of the estimated variances of the principal components at time  $t$ .

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Matrix  $C_t$  is an approximate correlation matrix for the original variables at time  $t$ . However, there is no guarantee that the elements on the diagonal of  $C_t$  are equal to 1, as we would like them to be.

### Step 6

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Standardize  $C_t$  so that it is a correlation matrix with all of its diagonal elements equal to 1. This is the same operation as taking a variance matrix into a correlation matrix. Call the new matrix  $\tilde{C}_t$ .

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### Step 7

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Scale  $\tilde{C}_t$  with the estimated variances of the original GARCH models for the variables at time  $t$  to get a variance matrix for the variables at time  $t$ .

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## 3 Prediction and Simulation

The ability to provide predictions is one of the key features of GARCH. Prediction for PC-GARCH is straightforward. Predict the principal component models and mimic steps 5 and 6, predict the variable models and mimic step 7.

Simulation is also a powerful tool, as [Burns, 2002] shows. Simulation with PC-GARCH is more problematic. While it may be possible to develop a simulation technique, it doesn't appear easy to produce one that both respects the GARCH processes of the variables and gets the correlations (and changes in correlation) right.

## 4 Discussion

The Orthogonal GARCH model of [Alexander, 2000] is in some respects similar to PC-GARCH. It also starts with a standardization of the original return data, but the standardization uses one mean and one standard deviation for each variable. A principal component rotation is performed on the standardized returns, and GARCH models are fit on the most important principal components.

Hence Orthogonal GARCH is less computationally intense than PC-GARCH. This is unlikely to be a good reason to prefer Orthogonal GARCH. In the mid-90's PC-GARCH was found to be feasible for problems containing a few hundred variables. With the increase in computing power since then, a problem would need to have several thousand variables before computing time became a serious issue.

A deliberate feature of PC-GARCH is that the variance for each variable is the same in PC-GARCH as the univariate model for the variable. A problem with Orthogonal GARCH is that this need not be true, and at times the variances can be substantially different. The down-side of PC-GARCH mimicking the univariate models is that there is no chance of information from other variables improving the estimate. In my experience with BEKK models the amount of such improvement seems to be small at best. It appears more likely in BEKK that precision of the variances is compromised in order to improve the covariances.

The principal components are uncorrelated over the whole sample period. However, PC-GARCH (and Orthogonal GARCH) want the principal components to be uncorrelated at each point in time. This is not the case. The adjustment that is made in step 6 may (or may not) alleviate to some extent the bad effects of this assumption.

The model of [Harris et al., 2004] is substantially different from PC-GARCH and Orthogonal GARCH. For each pair of variables it estimates four univariate GARCH models—one for each variable, one on the sum of the variables and one on the difference of the variables. The difference of the latter two yields 4 times the covariance between the two variables. This makes no assumptions about orthogonality. On the downside, though, it will not necessarily result in positive semidefinite estimates of variance matrices. For bivariate data this is unlikely to be a problem unless the variables are very highly correlated.

PC-GARCH has been used in several real-world settings. In no case was it found to be seriously defective. Although PC-GARCH probably improves relative to “real” multivariate GARCH models as the number of variables increases, it seems to work reasonably well even for bivariate and trivariate problems. An experiment was done to see if not performing the GARCH estimates for the smallest principal components would be a good thing. The results tended to support the notion that models for all principal components should be estimated. However, very small principal components can obviously be skipped with little loss.

All GARCH models want a lot of data. Simulations (both univariate and multivariate) have shown that 1000 observations is small—fewer than this and there is not going to be much real signal picked up. Even 5000 observations is not a very large sample in terms of the accuracy with which parameters are estimated. In practical terms this means that GARCH models require several years of daily data in order to be trustworthy.

It would be interesting to see comparison tests of these three techniques with financial data. The comparisons should be on out-of-sample predictions—in-sample comparisons hold little if any information.

## References

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